

Oct 19, 2007  
 Signals & Systems  
 Problem 1 Solution

$$\frac{\text{RUBRIC}}{(a) + (b) + (c)} = \boxed{25}$$

(1)

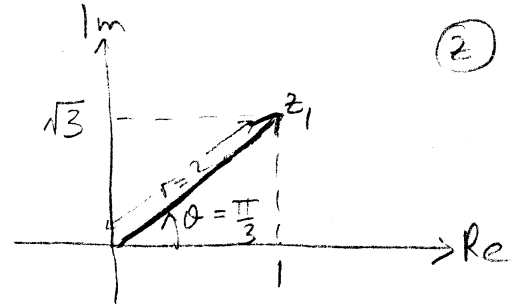
(a) Complex numbers

(i)  $z_1 = 1 + j\sqrt{3}$

3  $r = |z_1| = \sqrt{1^2 + \sqrt{3}^2} = 2$   
 $\theta = \angle z_1 = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60^\circ = \frac{\pi}{3} \text{ rad.}$

2/point each for magnitude, angle, sketch

$$z_1 = 2 e^{j\pi/3}$$



(2)

(ii)  $z_2 = \frac{e^{j\pi/3} - 1}{1 + j\sqrt{3}} = \frac{e^{j\pi/3} - 1}{2 e^{j\pi/3}} = \frac{1}{2} - \frac{1}{2} e^{-j\pi/3}$

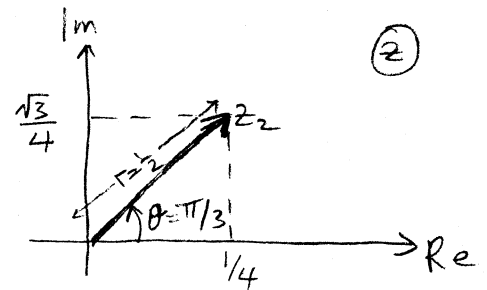
$$= \frac{1}{2} - \frac{1}{4} (1 - j\sqrt{3})$$

$$= \frac{1}{4} + j\frac{\sqrt{3}}{4}$$

3  $r = |z_2| = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2} = \frac{1}{2}$

$$\theta = \angle z_2 = \tan^{-1}\left(\frac{\sqrt{3}/4}{1/4}\right) = 60^\circ = \frac{\pi}{3} \text{ rad}$$

$$z_2 = \frac{1}{2} e^{j\pi/3}$$

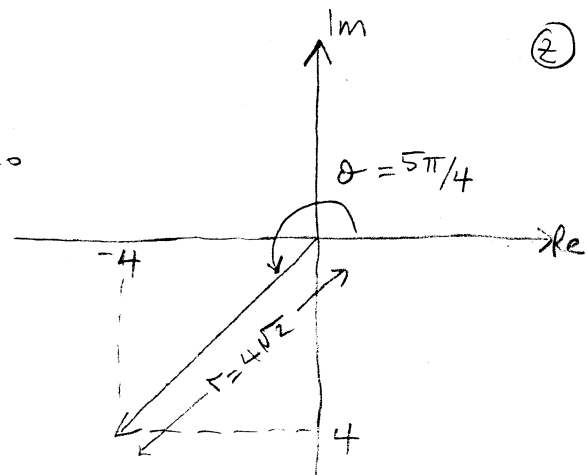


(2)

(iii)  $z_3 = (1 + j)^5 = (\sqrt{2} e^{j\pi/4})^5$   
 $= (\sqrt{2})^5 e^{5\pi/4} = 4\sqrt{2} e^{5\pi/4}$

3  $r = |z_3| = 4\sqrt{2}$ ,  $\theta = \angle z_3 = \frac{5\pi}{4} \text{ rad} = 225^\circ$

$$z_3 = 4\sqrt{2} e^{5\pi/4}$$



(2)